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EVIDENCE OF SPALL FROM DEGHOSTING OF SHORT-PERIOD TELESEISMS

W. C. Dean and J. H. Goncz

Seismic Data Analysis Center

Teledyne Geotech, 314 Montgomery Street, Alexandria Virginia 22314

12 April 78

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Spall signals, if present on teleseisms, should be most easily detected in the short-period P-waveforms. If pP and spall (P_s) echoes are identical in waveform to the P then, theoretically, deghosting filters could remove these source echoes and produce identical seismograms from neighboring explosions with nearly identical source-to-receiver paths. If P_s is absent, then pP corrections alone suffice.

Determining the optimum echo amplitude and delay parameters involves trial-and-error and using known event depths and close-in measurements of pP and

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P_s echo delays when available.

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Optimum deghosting for pP echoes improved the correlation and deghosting both pP and P_s echoes improved the correlations and waveform match still more. Moreover, just as expected, the polarity for the optimum pP echoes was always negative and for the P_s echoes, positive. Spall echoes ranged between 0 and 35% of the P amplitude. In addition, adjustments for variations in corner frequency between pairs of events seem necessary to achieve matching P waveforms.

Seismograms deghosted with these optimum echo parameters never achieved the correlation levels of KNICKERBOCKER and CHATEAUGAY. Furthermore, the optimum echo parameters found at one station did not always increase the correlations between the same event pairs at other stations. Evidence exists that both the pP and P_s echoes are lowpass filtered verisons of the P, rather than exact copies. This situation may exist because of topographical scattering for pP and a time distribution of the return of spall material for P_s.

Although much of the analysis used data from RKON at a distance of 21° from NTS, most event pairs of interest had distances to RKON which were equal to within 1-3 km. Thus, we feel that possible complications due to upper mantle triplications are not a problem for this study.

EVIDENCE OF SPALL FROM DEGHOSTING OF SHORT-PERIOD TELESEISMS

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ABSTRACT

Spall signals, if present on teleseisms, should be most easily detected in the short-period P-waveforms. If pP and spall (P_s) echoes are identical in waveform to the P then, theoretically, deghosting filters could remove these source echoes and produce identical seismograms from neighboring explosions with nearly identical source-to-receiver paths. If P_s is absent, then pP corrections alone suffice.

Determining the optimum echo amplitude and delay parameters involves trial-and-error and using known event depths and close-in measurements of pP and P_s echo delays when available.

Two explosions at Pahute Mesa, KNICKERBOCKER in 1967 and CHATEAUGAY in 1968, which were within 25 meters of the same depth and within 0.5 kilometers of the same location, generated virtually identical seismograms at all stations; their correlation coefficient exceeded 0.95 at RKON. For other neighboring pairs of Pahute Mesa explosions correlation coefficients ranged between 0.50 and 0.70.

Optimum deghosting for pP echoes improved the correlation, and deghosting both pP and P_s echoes improved the correlations and the waveform match still more. Moreover, just as expected, the polarity for the optimum pP echoes was always negative and for the P_s echoes, positive. Spall echoes ranged between 0 and 35% of the P amplitude. In addition, adjustments for variations in corner frequency between pairs of events seem necessary to achieve matching P waveforms.

Seismograms deghosted with these optimum echo parameters never achieved the correlation levels of KNICKERBOCKER and CHATEAUGAY. Furthermore, the optimum echo parameters found at one station did not always increase the correlations between the same event pairs at other stations. Evidence exists that both the pP and P_s echoes are lowpass filtered versions of the P, rather than exact copies. This situation may exist because of topographical scattering for pP and a time distribution of the return of spall material for P_s.

Although much of the analysis used data from RKON at a distance of 21° from NTS, most event pairs of interest had distances to RKON which were equal to within 1-3 km. Thus, we feel that possible complications due to upper mantle triplications are not a problem for this study.

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INTRODUCTION

Most underground nuclear explosions produce spallation. The surface layer of rock torn loose by the initial upward traveling shock wave falls to earth again within from one to three seconds. Thus, the P-waveform complex could be composed of the path filtered effects of at least three impulses generated near the source: one from the initial P-wave, a second from the pP reflection, and a third from the spallation debris falling back.

The objective of this study is to determine whether some evidence of spall can be detected on teleseisms. To date, only meagre evidence of spall on teleseismic records exists. Based upon close-in accelerometer measurements Viecelli (1973) estimated theoretically that spallation might generate a significant increase in the magnitude of surface waves. Von Seggern (1973) working with records from Amchitka events found no appreciable effect on M_s measurements, but he did see some evidence of surface wave delay. Springer (1974) observed what he thought were secondary P-wave arrivals in short-period teleseisms from Pahute Mesa explosions. Cohen (1975), using cepstral analysis, thought that either pP or P_s (spall) but not both might be predominant secondary arrivals at teleseismic stations. Using subsurface accelerometer data, Sobel (1977) estimated that little, if any, spall energy left the source region and, therefore, spall energy would have negligible effect on m_b and M_s.

Viecelli, J. A. 1973. Spallation and the generation of surface waves by an underground explosion, J. Geophys. Res., 78, 2475-2487.

von Seggern, D., 1973. Seismic surface waves from Amchitka Island test site events and their relation to source mechanism, J. Geophys. Res., 78, 2467-2474.

Springer, D. L., 1974. Secondary sources of seismic waves from underground nuclear explosions, Bull. Seism. Soc. Am., 64, 581-594.

Cohen, T. J., 1975. P_s and pP phases from seven Pahute Mesa events, Bull. Seism. Soc. Am., 65, 1029-1032.

Sobel, P. A., 1977. The effects of spall on m_b and M_s, SDAC-TR-77-12, Teledyne Geotech, Alexandria, Virginia.

THE DEGHOSTING APPROACH

P-waves are the most easily detected phases at teleseismic stations. Other phases, normally smaller than P, are in evidence in the P coda. These overlapping phases, such as pP, frequently can be separated from the P by deghosting. Hence, detailed analysis or decomposition of the P-wave, and its coda, offers hope of exhibiting spall signals on teleseisms.

Spall signals, if they are in evidence at teleseismic stations, should echo the original P-wave signal in a way similar to the pP echo. The pP reflection closely approximates the direct P-wave signal in waveform, and it is normally of opposite polarity. Also, it occurs with a delay of one second or less from nuclear explosions buried at less than 5000 feet, and it has an amplitude somewhat smaller than the initial P-wave. The spall signal should be an echo of the P-waveform similar to the pP echo. The spall echo should differ from the pP because its arrival would be expected later, one to three seconds after the P-wave arrival, and its polarity should be the same as P since the material striking the earth should cause a compression.

Although deghosting filters (Sax, 1967) can readily remove an echo such as pP from a P-waveform, with a single seismogram no measure exists of what the waveform of the unechoed P-phase through that source-to-receiver path is like. Thus, pairs of events must be found where the source-to-receiver paths agree. Then, by correcting for pP and spall echoes on differing seismograms from different events, P-waveforms which agree may be generated. If so, the echo model for spall signals will be justified, spall echoes possibly detected, and some estimate gained of their size.

To test this hypothesis the following comparison is sought:

a. one or more pairs of events with nearly the same locations and depths which yield nearly identical P-wave signals at teleseismic stations. For such events the paths agree so the P-waves agree, the pP echoes agree, and the spall signals are either not present or agree.

Sax, R. L., 1967. Noise analysis of single channel deghosting filters, Seismic Data Laboratory Report 178, Teledyne Geotech, Alexandria, Virginia. AD 810792

b. one or more pairs of events with the same location and different depths. For these events, pP echoes are different. Deghosting traces of pP should make them agree, unless a spall signal is present.

c. one or more pairs of events at similar depths, but different locations, for which the P-waveforms do not agree. The effects of source separation (path differences) account for the waveform mismatch. With several such pairs it may be possible to quantize the P-waveform mismatch versus source separation.

If nearby events with mismatching P-waveforms at a common station can be deghosted to fit assuming only a pP echo, then the spall echo is not detectable at that station. If however, a spall echo is required to achieve a high correlation between deghosted waveforms, then the spall signal has been detected.

If a P-waveform fit is established by compensating for both pP and spall echoes on a single pair of events, then the primary objective of this study will be accomplished. Success is not needed on all pairs of nearby events because the P-waveforms can disagree due to varying strain release as well as other reasons.

TWO EVENTS WITH MATCHING SIGNALS

The hypothesis being tested to detect evidence of spall on teleseismic records is that explosions of nearly equal depths and epicenters should provide similar seismograms at the same stations provided no, (or equal), spall is present. If the events have different depths, then correcting for pP echoes alone ought to produce like seismograms, provided there is no (or equal) spall effect.

Two Pahute Mesa explosions whose epicenters are within 0.5 kilometers of each other do give nearly identical seismograms at teleseismic stations.[1] They are KNICKERBOCKER (2069 feet deep) and CHATEAUGAY (1992 feet deep). Figure 1 shows recordings of these events at four stations: RKON, WH2YK, NPNT, and HNME. The signal waveforms vary appreciably from station to station but for these two events they are remarkably similar at the same stations. The correlation coefficient (xmax) is shown between the pair of seismograms for each site.[2] The peak crosscorrelations range between 86 and 96 percent for all sites except HNME.

The lower correlation at HNME for these two events is a result of poorer signal-to-noise at that site. Figure 2 shows these two HNME seismograms for the raw signals (upper two traces) and then the same two signals filtered with a bandpass (phaseless) filter from 0.5 to 3.0 Hz with 24 db/octave cutoffs. The xmax readings improve from the 59 percent on the raw recordings to 81 percent on the filtered recordings where much of the background noise has been removed. Thus, KNICKERBOCKER and CHATEAUGAY events demonstrate that underground explosions at virtually the same depth and location do yield almost identical seismograms at the same teleseismic stations.

Table I lists the correlation coefficients at RKON for several pairs of events at Pahute Mesa for various depths and horizontal separations. The

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1. See Springer and Kinnaman (1971) and (1975) for all yield, depth, and location data on the events discussed in this report.

2. The correlation coefficient is equal to the maximum of the crosscorrelation function which is abbreviated as xmax in this report. The seismograms shown are comprised of 1024 samples at 20 samples per second or 51.2 seconds of data. In this report all crosscorrelations use only the first half (512 points) with approximately 21 seconds of signal and 4.6 seconds of noise prior to the signal.

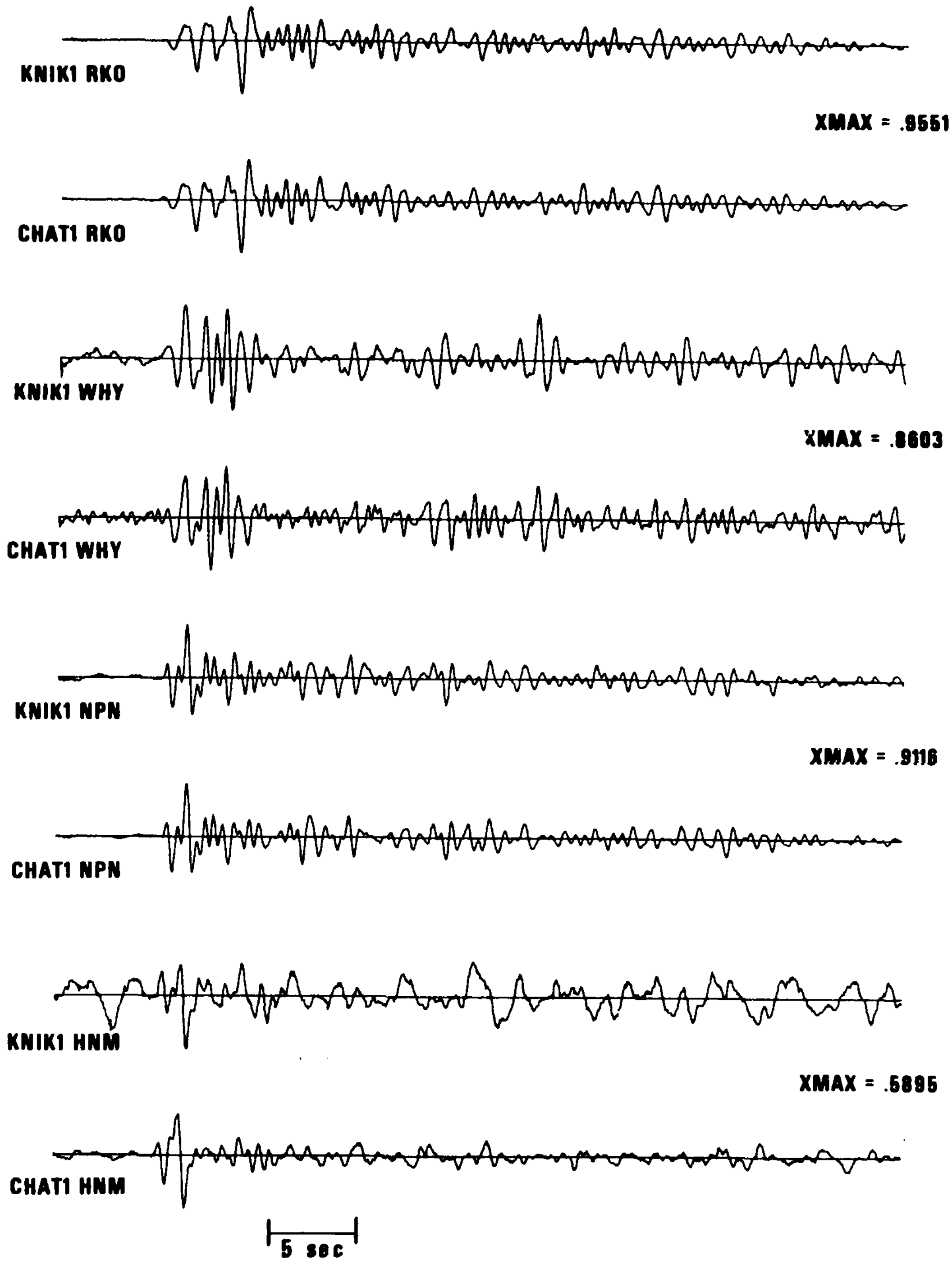

Figure 1. Raw signals from KNICKERBOCKER and CHATEAUGAY at four different stations. The correlation coefficient (xmax) is indicated between the pairs of signals for each site.

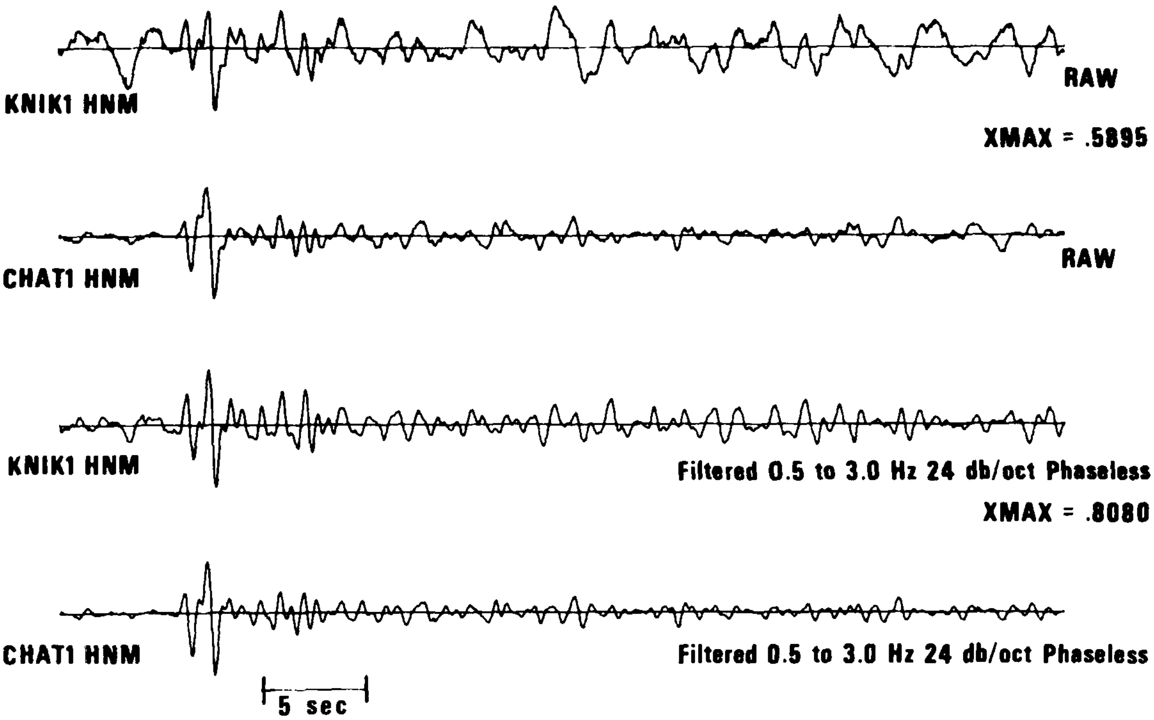

Figure 2. KNICKERBOCKER and CHATEAUGAY at HNME. The first two traces are the unfiltered seismograms at HNME. The third and fourth are these two seismograms filtered 0.5 Hz to 3.0 Hz with 24 db/octave cutoffs.

TABLE I

Correlation Coefficient Between Events of Similar Depths

EVENT	DEPTH feet	Δ to RKON km	Δ to RKON degrees	EVENT SEPARATION r(km)	δΔ (km)	XMAX
KNICKERBOCKER	2069'	2350	21.14	0.5	.8	.960
CHATEAUGAY	1992'	2351	21.15			
CHATEAUGAY	1992'	2351	21.15	4.7	3	.579
PURSE	1964'	2348	21.13			
POOL	2884'	2340	21.05	5.5	5	.716
INLET	2687'	2345	21.10			
POOL	2884'	2340	21.05	7.5	7	.759
HALFBEAK	2688'	2333	20.99			
FONTINA	3999'	2348	21.13	8.8	1	.461
CHESIRE	3829'	2347	21.11			
KASSERI	2060'	2342	21.08	8.9	8	.611
KNICKERBOCKER	2069'	2350	21.14			
KASSERI	2060'	2342	21.08	9.8	4	.653
ESTUARY	2851'	2346	21.10			
INLET	2687'	2345	21.10	12.7	12	.746
HALFBEAK	2688'	2333	21.00			
POOL	2884'	2340	21.05	19.7	6	.633
ESTUARY	2851'	2346	21.10			

range of NTS to RKON, the station recordings on which most of these examples were computed, is 21°. At this range the recorded signal is complicated by triplications and caustics (see Helmberger and Wiggins, 1971). Such signal complexities are both advantageous and disadvantageous. The advantage is that the more complex the P-waveform from a single impulsive source is, the better the deghosting method will work in removing source echoes from pP and spall. The disadvantage is that slight changes in epicentral distance cause a large change in signal waveform due to the path complexities. Thus pairs of events must be closer together to correlate well. It is for this reason that the tables show for RKON examples, the epicentral range, Δ, and the change in range, $\delta\Delta$, as well as the total event separation, r. Two events could be separated by several kilometers and still be at nearly the same epicentral distance from RKON. Note in particular that both the lowest and highest correlations occur for event pairs whose distances from RKON are equal to better than 1 kilometer, indicating that for these small differences in epicentral distance the waveshape is not changing due to distance to the station as much as due to distance between events.

These correlations range from 46 to 96 percent. In all cases the signal-to-noise ratio for these RKON seismograms was good. The correlation coefficients for the event pairs listed in Table I do not show a simple tendency to decrease as the separation between events becomes larger.

In contrast, Table II shows the correlation coefficients between events mismatched in depth and station and for the same event at different stations. These crosscorrelations range from 21 to 35 percent, when both the sources and stations differ. For one event (CAMEMBERT), recorded at different stations (RKON and WH2YK), the correlation coefficient was 56 percent. We find that the correlations (xmax's) generally agree with these specific examples: when the source-to-receiver path agrees, the correlation between explosion events is high, but when the source-to-receiver path disagrees, then the correlation between events is low.

Helmberger, D. and Wiggins, R. A., 1971. Upper mantle structure of Midwestern United States, J. Geophys. Res., 76, 3229-3245.

TABLE II

Correlation Coefficients Between Events
Mismatched in Depth, Station, and Source Location

EVENT	DEPTH	STATION	Δ	EVENT SEPARATION r(km)	XMAX
BENHAM	4600'	RKON	21°	6.4	.217
REX	2204'	NPNT	39°		
MAST	2989'	RNWV	29°	17.1	.351
DURYEA	1786'	RKON	21°		
MUENSTER	4764'	HNME	37°	17.8	.205
TYBO	2510'	WH2YK	26°		
CAMEMBERT	4301'	RKON	21°	-	.558
CAMEMBERT	4301'	WH2YK	26°		
CAMEMBERT	4301'	HNME	37°	-	.346
CAMEMBERT	4301'	FNWV	29°		

DEGHOSTING pP ECHOES

When two events have neighboring epicenters, though at different depths, their teleseismic P-waveforms correlate well though appreciably less than the KNICKERBOCKER-CHATEAUGAY pair that had nearly identical epicenters and depths. A suite of seven events, including KNICKERBOCKER and CHATEAUGAY, were separated from each other as shown in Table III. Their correlation coefficients (xmax's), together with their SP-Z signals recorded at RKON are shown on Figure 3. The correlations (xmax) are compared with FONTINA, the event whose epicenter is most central to the group.

The correlations with FONTINA that these RKON signals exhibited (ranging from .40 to .74 with an average of .50) should improve if the pP reflections were removed, a move that synthetically places all events at zero depth. Deghosting the signals of pP, especially if there are no spall reflections or other source differences, should achieve agreement in the teleseismic waveforms.

Figure 4 shows these waveforms when all seismograms have been deghosted, assuming a velocity of 3.9 km/sec and pP reflection coeffients of -0.6. The corresponding correlations relative to the FONTINA event are shown at each trace. Observers can visually notice a significant increase in the similarity of the signal waveforms. In addition, the correlations increased, although not markedly, to a new average of .52.

The shallower events still appear to contain a high frequency that the deeper ones did not exhibit. If this difference is attributed to variations in the source corner frequency, then a low pass filtering of the seismograms from the shallower events ought to improve the fit even further. Figure 5 shows the results of the (pP) deghosting plus the corner frequency filtering. No quantitative estimates were made of what the proper corner frequency should be. Rather, the low frequency cutoff (6 db/octave) was varied until the waveform fit appeared to be visually best. However, the cutoff for the corner frequency filter was always lower for the shallower of any pair of events. Now, all seismograms in this set have similar waveforms and therefore appear to belong to the same family of events. The correlations (xmax's) have increased to an average of .57. Table IV summarizes these RKON statistics.

TABLE III

Horizontal Displacements between the Epicenters of Eight Neighboring Pahute Mesa Events.

Event	depth (ft)	PURSE	CHAT	KNIK	ESTU	BOX	FONT	COLB	BEN
				in kilometers					
PURSE	1964'	-							
CHATEAUGAY	1992'	4.7	-						
KNICKERBOCKER	2069'	4.5	0.5	-					
ESTUARY	2851'	3.0	7.4	7.2	-				
BOXCAR	3800'	5.2	6.3	5.9	5.0	-			
FONTINA	3999'	2.0	2.9	2.7	4.5	4.4	-		
COLBY	4177'	4.2	6.9	6.5	3.0	2.1	4.3	-	
BENHAM	4600'	6.5	1.8	2.0	9.2	7.4	4.7	8.3	-

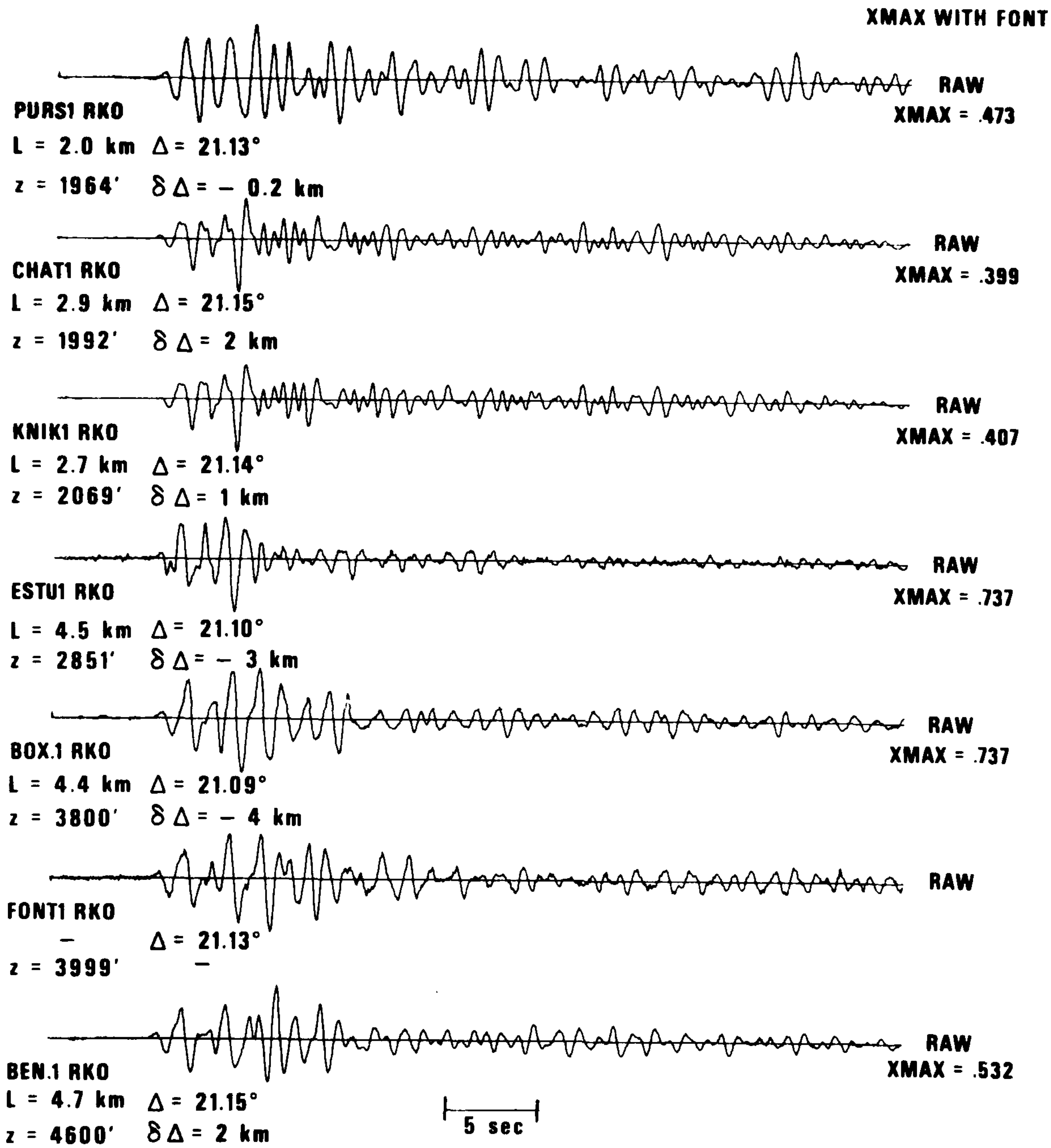

Figure 3. Seven neighboring Pahute Mesa events as recorded at RKON. Listed are their correlation coefficients (xmax's) versus the FONTINA event, their lateral displacements (L) from the FONTINA epicenter, their depth (z), and their relative distance to RKON with respect to FONTINA.

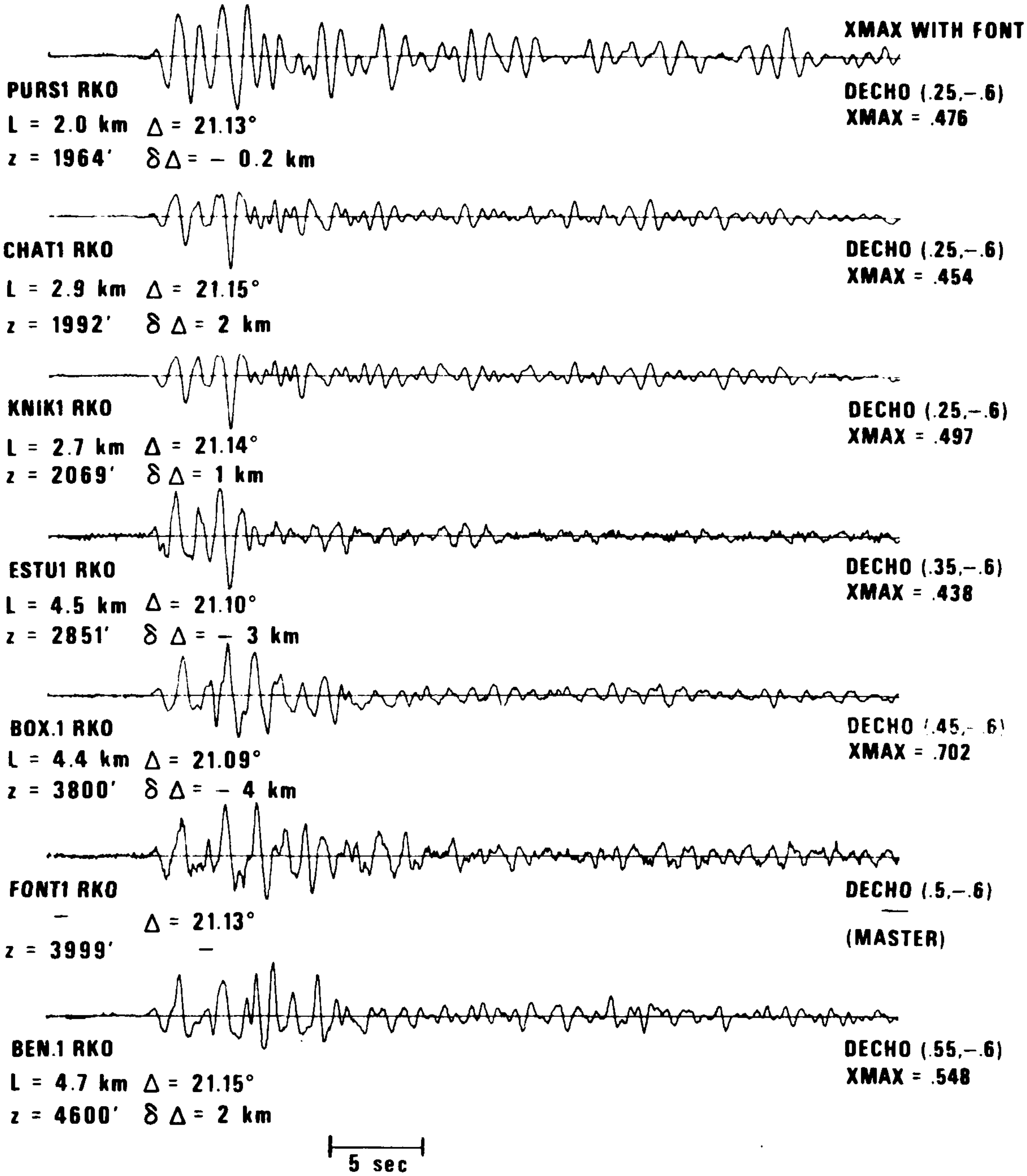

Figure 4. The same seven Pahute Mesa events at RKON with the signals DECHO'd assuming a velocity of 3.9 km/sec and a pP reflection coefficient of −0.6.

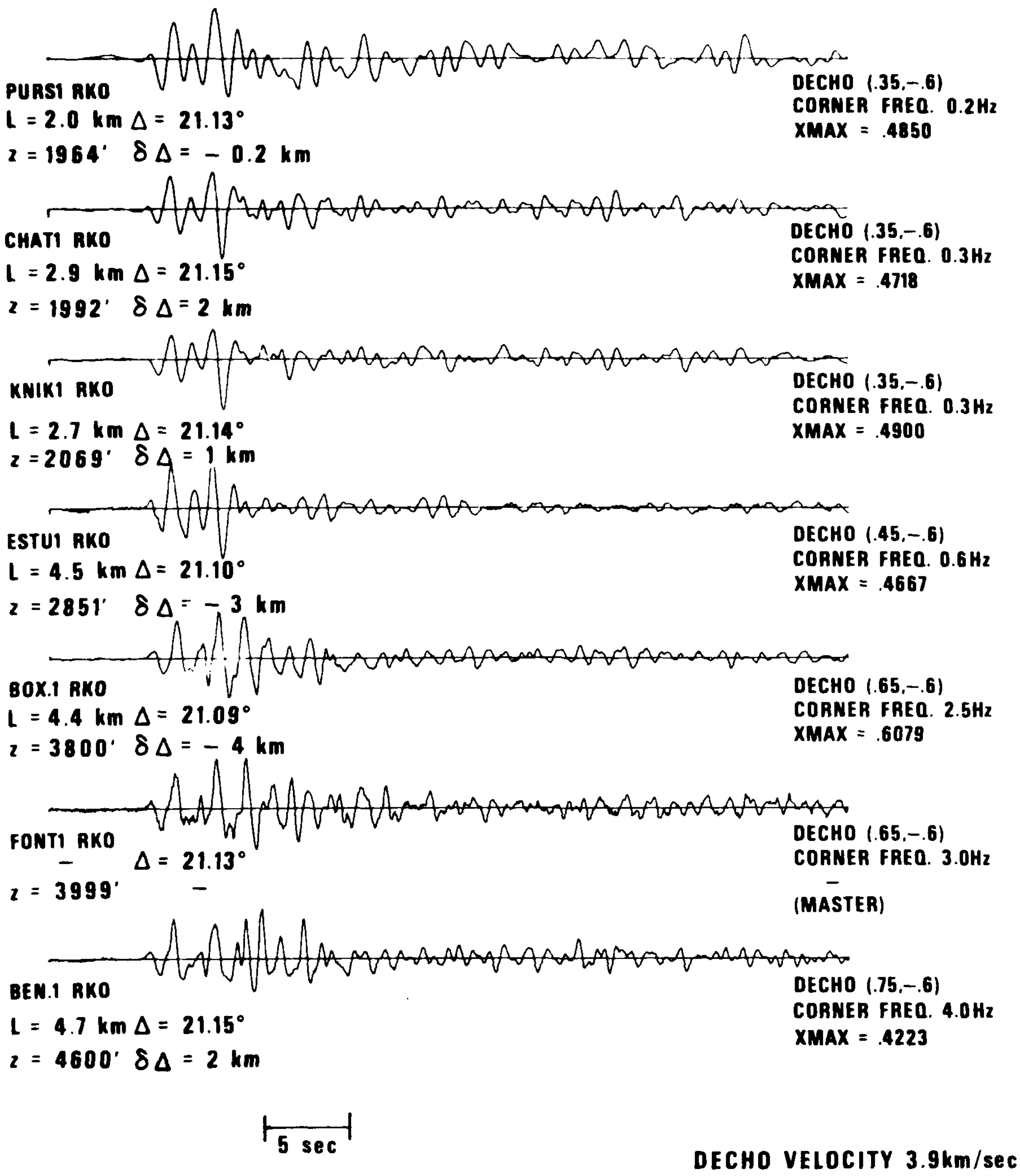

Figure 5. These signals were DECHO'd (3.9 km/sec velocity, -0.6 ampl.) and adjusted for corner frequency. The event correlations are computed versus FONTINA.

TABLE IV

Raw and Deghosted Correlations at RKON and HNME for Eight Neighboring Events

RKON vs. FONTINA: crosscorrelation (xmax)

Event	Depth	raw	decho pP	decho pP & corner freq.	r(km)	δΔ (km)
PURSE	1964	.473	.476	.534	2.0	-0.2
CHATEAUGAY	1992	.399	.454	.524	2.9	2
KNICKERBOCKER	2069	.407	.497	.554	2.7	1
ESTUARY	2851	.468	.438	.503	4.5	-3
BOXCAR	3800	.737	.702	.724	4.4	-4
FONTINA	3900	----	----	----	----	----
BENHAM	4600	.532	.548	.557	4.7	2
average		.503	.519	.566		

HNME vs. FONTINA: crosscorrelation (xmax)

Event	Depth	raw	decho pP	decho pP & corner freq.	r(km)
PURSE	1964	.465	.455	.585	2.0
CHATEAUGAY	1992	.512	.502	.494	2.9
KNICKERBOCKER	2069	.312	.381	.394	2.7
ESTUARY	2851	.550	.584	.640	4.5
BOXCAR	3800	.590	.609	.632	4.4
FONTINA	3999	----	----	----	----
COLBY	4177	.729	.709	.726	4.3
BENHAM	4600	.614	.599	.610	4.7
average		.539	.548	.583	

If this pP deghosting and corner frequency filtering procedure is correct, then the same operations with corresponding parameters ought to yield similar improvements at other stations. These same events recorded at HNME are shown in Figure 6, and the corrected seismograms (both pP deghosting and corner frequency filtering) are shown in Figure 7. Again the correlations (xmax's) improve from an average of .54 on the original signals to an average of .58 on the set corrected for both pP and the corner frequency. Extra bandpass filtering was required for KNICKERBOCKER because the signal-to-noise ratio, already poor on the original seismograms at HNME, was made worse by the deghosting and corner frequency filtering.

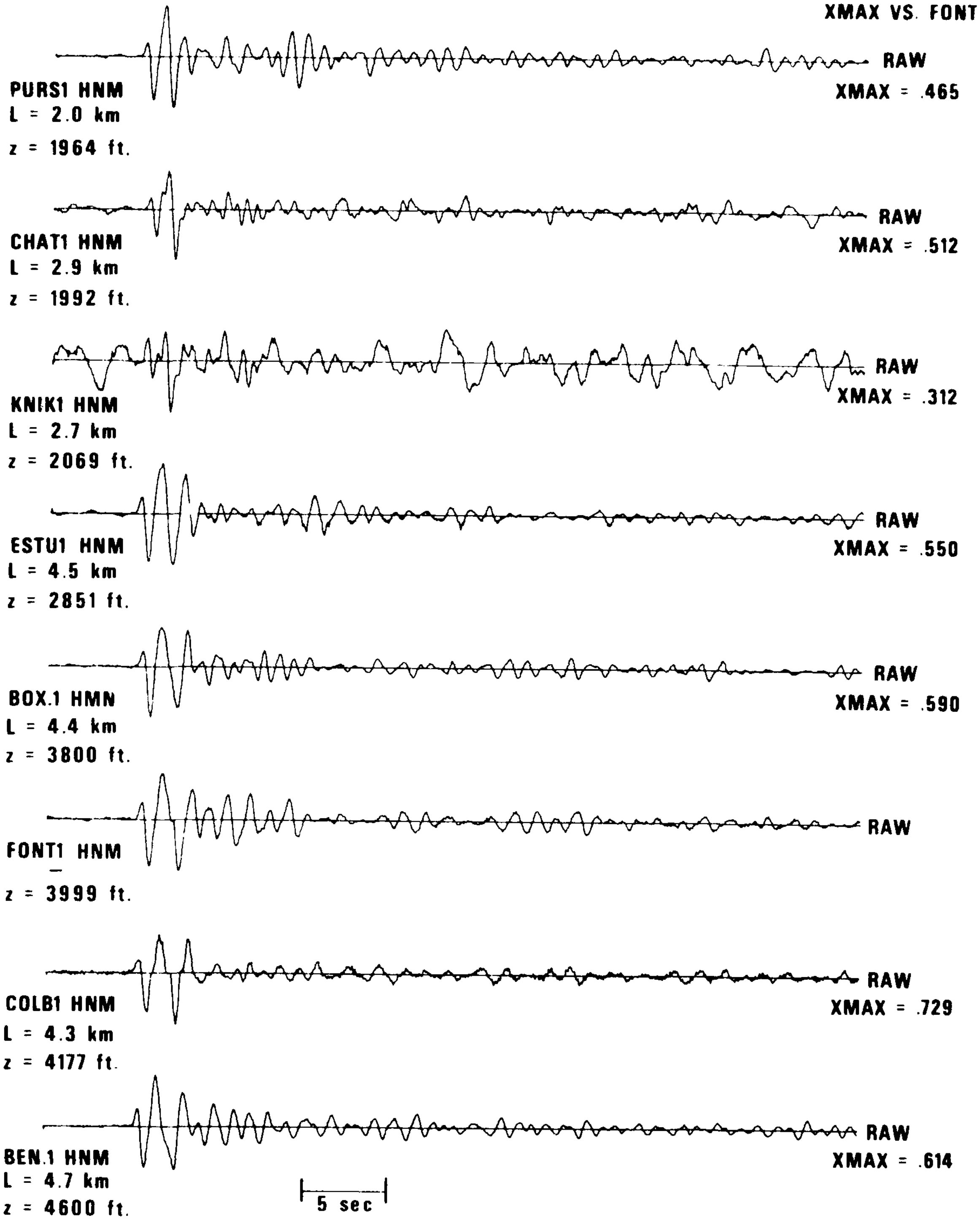

Figure 6. Eight neighbor ng Pahute Mesa events as recorded at HNME. Listed are their xmax's versus FONTINA, their lateral displacement (L) from the FONTINA epicenter, and their depth (z).

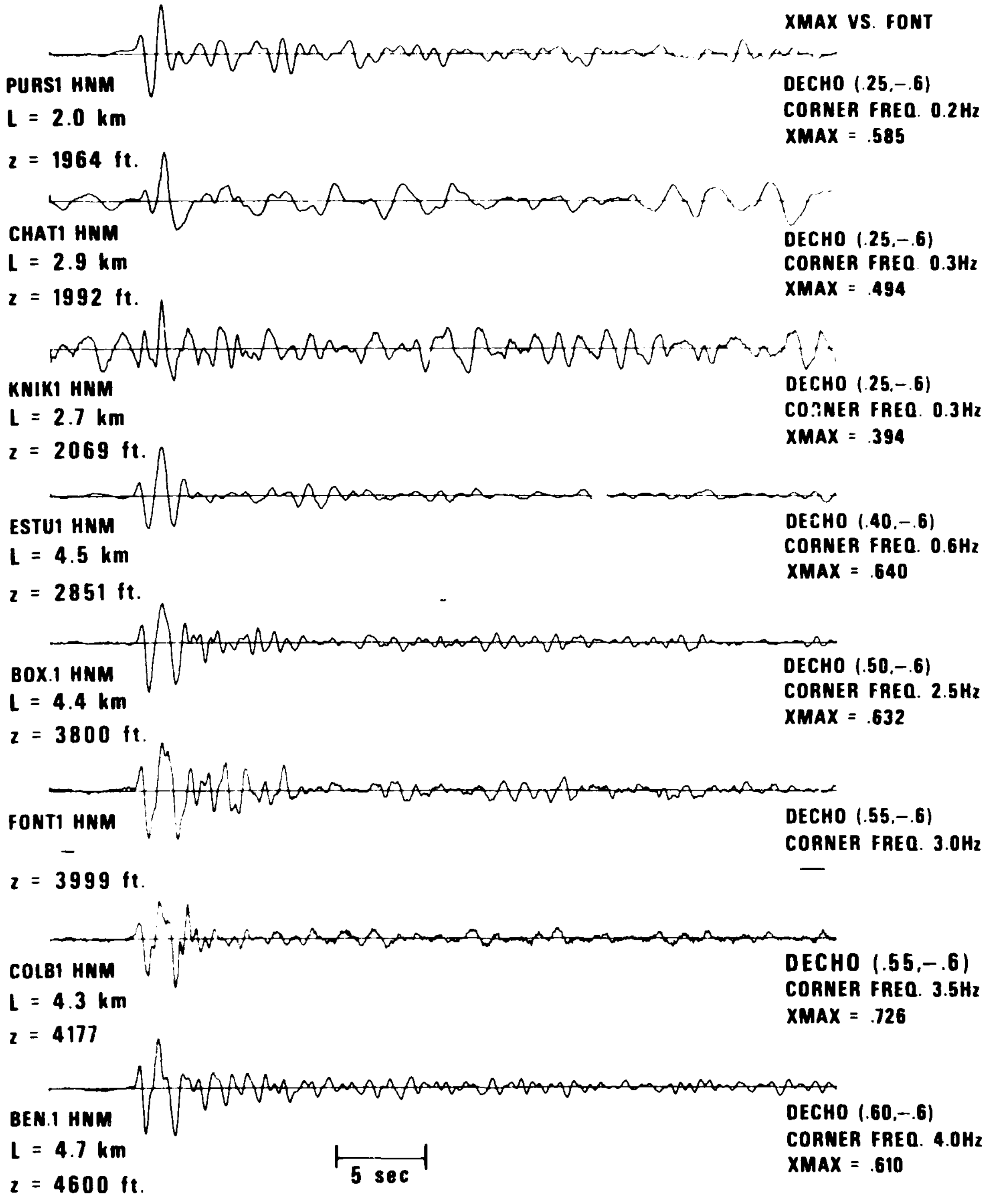

Figure 7. These signals were DECHO'd (3.9 km/sec velocity & -0.6 ampl.) and adjusted for corner frequency filtering.

SYNTHETIC SPALL ECHOES

Correcting the eight neighboring events at Pahute Mesa for pP and corner frequency differences of the sources improves the similarity of the first five seconds of P-wave for both the RKON and HNME records. Although the correlation coefficients (xmax's) improved, they did not achieve the level of the correlations with no corrections between the KNICKERBOCKER and CHATEAUGAY signals. One explanation for this situation is that the differences still remaining after corner frequency and pP corrections in the Pahute-to-RKON seismograms are due to spall signals. Supporting this argument is the long coda occurring on the seismograms from the deeper events that have high yields. The question then arises of how much effect a spall echo would cause on the waveform and the correlation (xmax) measurements.

To demonstrate this effect the KNICKERBOCKER and CHATEAUGAY signals can be used with spall echoes added to them. Since these seismograms come from different events, but are so much alike (xmax = .955), they can be assumed to represent the same P-waveform from two events with the pP reflection removed. Following our model this P-waveform is added to itself with the same polarity, and delayed a time appropriate to spall echoes. Crosscorrelating the unechoed seismogram with the synthetic spall-echoed version from the other event will show how much effect varying amounts of spall echo have in both correlation coefficient and waveform appearance.

Figure 8 shows the results of this test. The first two traces are those recorded at RKON from KNICKERBOCKER and CHATEAUGAY. The third, fourth, and fifth traces show KNICKERBOCKER with 10%, 20% and 30% spall echoes respectively, each echo occurring 2.0 seconds behind the P. Trace six is the CHATEAUGAY signal with a spall-like echo of 20% added at 1.6 seconds behind the P.

The correlation coefficients between various pairs are also shown on Figure 8. These results are summarized in Table V.

This synthetic example shows that although spall echoes decrease the correlation between two signals, the reduction is not substantial and the echoed waveforms and the unechoed originals still look very much alike.

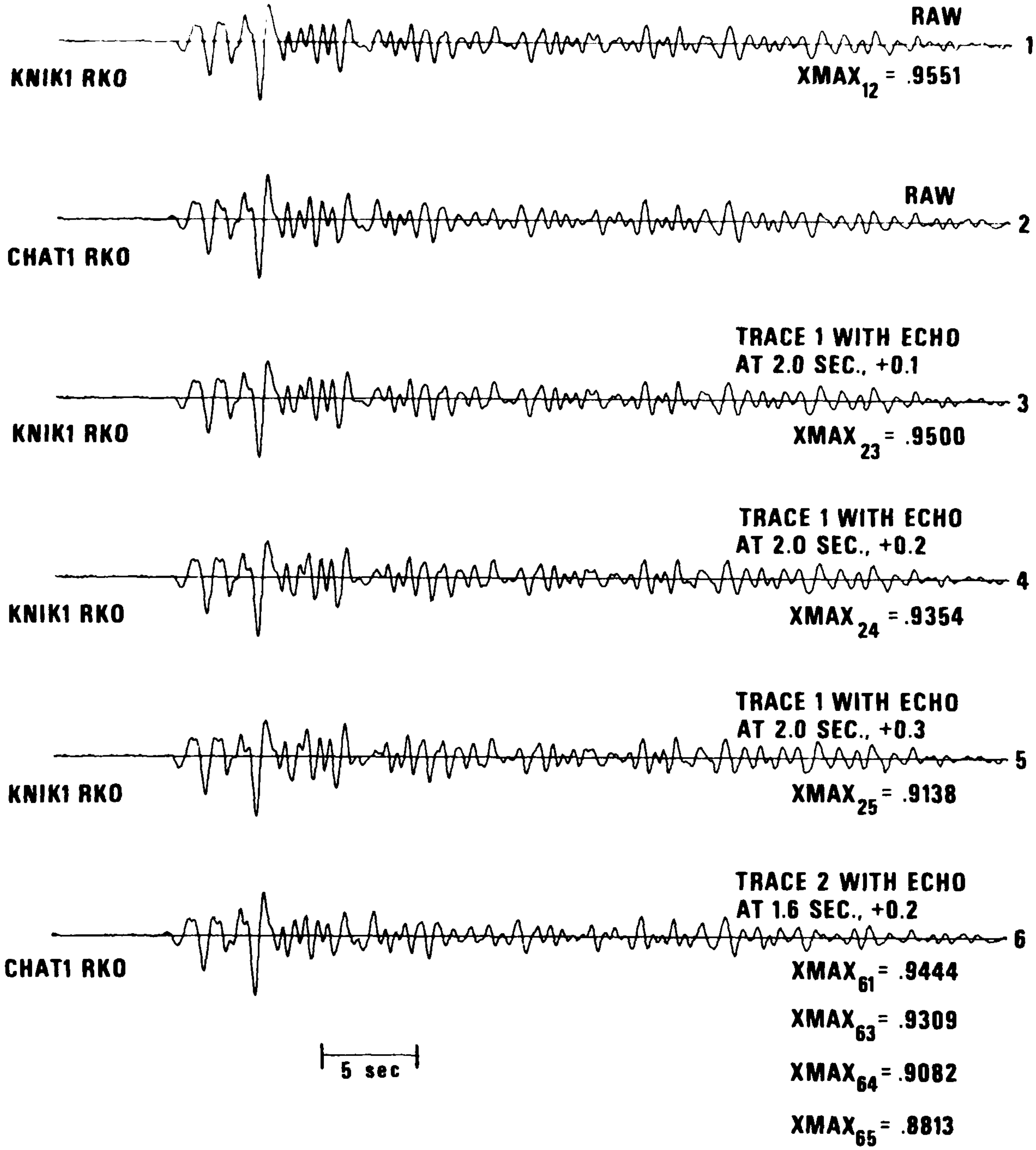

Figure 8. A synthetic test using KNICKERBOCKER and CHATEAUGAY seismograms at RKON showing the effects on waveform and xmax from adding 10%, 20%, and 30% spall echoes. The subscripted numbers on the crosscorrelations, e.g. $XMAX_{12}$, indicated the two traces (1 and 2) which have been crosscorrelated.

TABLE V

Decrease in Correlation with Synthetic Spall Echoes

RKON CHATEAUGAY unechoed vs. KNICKERBOCKER with spall echo at 2.0 secs.

Echo amplitude	Maximum crosscorrelation
0%	.955
10%	.950
20%	.935
30%	.914

CHATEAUGAY echoes at 1.6 seconds (20%) vs. KNICKERBOCKER with spall echo at 2.0 secs.

Echo amplitude	Maximum crosscorrelation
0%	.944
10%	.931
20%	.903
30%	.881

PUBLISHED pP and P_s TIMES

Published data from close-in measurements of spall delay times is sparse. Springer (1974) gives such readings for thirty-seven shots including six at Pahute Mesa. Three of these shots, which are in our data base are HALFBEAK, SCOTCH, and BOXCAR. The first three traces on Figure 9 show the SP-Z signals from each of these at RKON. Springer's table assumes a source-to-receiver distance of 50^{o} with a take-off angle of 25^{o}. At RKON (21^{o}) the take-off angle is 38^{o} and so Springer's pP times must be multiplied by $\cos 38^{o}/\cos 25^{o}$ (see Table VI). The last three traces on Figure 9 show the RKON seismograms deghosted using these pP and P_s delays. Echo amplitudes of -0.4 were assumed for pP and +0.2 for spall. In addition to the deghosting filter, the BOXCAR seismogram was highpass filtered (cutoff 1.0 Hz) to adjust for a different corner frequency from the other events. The deghosted traces show more similarities than did the originals. However, the average of the three correlations (Table VI) declined and only on one of the three pairs did it improve with deghosting. The source separation ranging between nine and eighteen kilometers for pairs of these events may be too large to achieve a good waveform match.

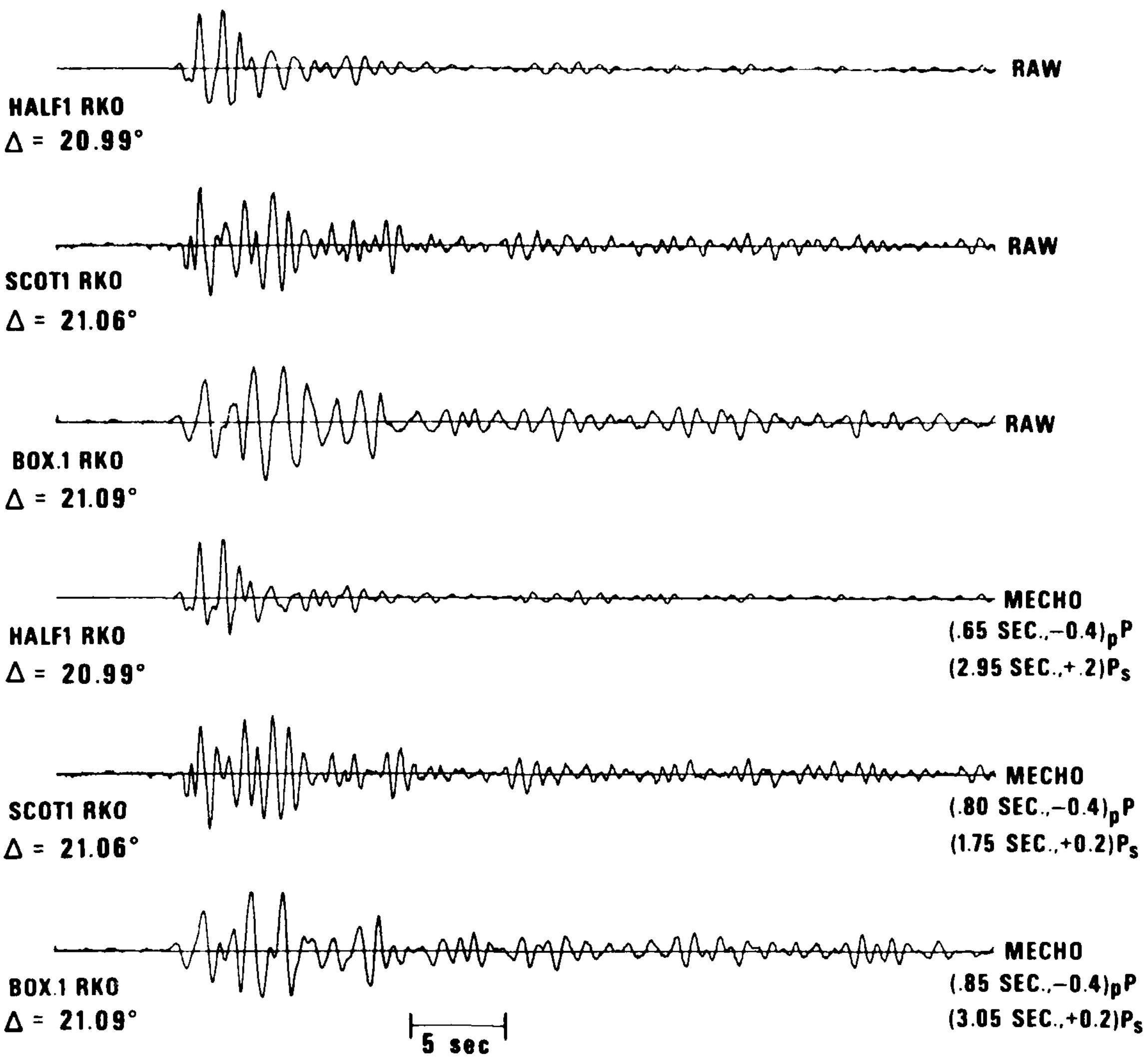

Figure 9. Raw and deghosted seismograms at RKON from three events for which close-in measurements are available for pP and P_s delay times.

TABLE VI.

HALFBEAK, SCOTCH, and BOXCAR Correlations at RKON.

Event	pP-P(sec) at 50^o	pP-P(sec) at 21^o(RKON)	P_s-P(sec)	Δ
HALFBEAK	.72	.63	2.93	20.99°
SCOTCH	.91	.79	1.73	21.06°
BOXCAR	.96	.83	3.07	21.09°

Correlation coefficients (xmax) at RKON

Event pair	xmax	processing
HALFBEAK vs. SCOTCH	.5240	raw signals
HALFBEAK vs. BOXCAR	.4804	raw signals
SCOTCH vs. BOXCAR	.4588	raw signals
HALFBEAK vs. SCOTCH	.5219	deghosted
HALFBEAK vs. BOXCAR	.3930	deghosted
SCOTCH vs. BOXCAR	.5029	deghosted

Echo amplitudes assumed -0.4 for pP (all events)
+0.2 for P_s (all events)

The original and deghosted traces shown on Figure 9.

A comparison of correlations for raw signals and deghosted (pP and P_s) signals between HALFBEAK, SCOTCH, and BOXCAR at RKON. Both pP and P_s echo times are estimated from close-in measurements on these three events.

DEGHOSTING pP and P_s BY OPTIMUM SEARCH

Neither the set of seven neighboring events (with separations up to five kilometers) on which pP-only corrections were applied nor the set of three events for which pP and spall delay measurements exist (with separations from nine to eighteen kilometers) have achieved a waveform match, after deghosting, that approximates the original signals from KNICKERBOCKER and CHATEAUGAY. If source separation is the problem, then work is needed on pairs which are closer. Another pair of events separated by only 0.3 kilometers, but at different depths, is MAST (at 2989 feet) and CHARTREUSE (at 2183 feet). With these events a deghosting search routine was followed in an effort to achieve a waveform match.

In general searches must be undertaken over an eight parameter space for an optimum fit: two echo delays and two echo amplitudes for each of two seismograms. Even if the search space is restricted to a pP-only fit, four parameters must be optimized. The procedure for a pP search could be to set three of the four parameters, vary the fourth over a range and compute the maximum of the crosscorrelation for each setting. For echo amplitude ranges from -0.8 to +0.8 and echo delay times from 0.1 to 1.5 seconds for pP and 1.0 to 3.5 seconds for P_s at sampling intervals of 0.05 in both amplitude and time, the test of every combination requires more computing time than available ($.9 \times 10^3$ pP tests times 1.5×10^3 P_s test times .25 minutes per test on the PDP-15). However, some shortcuts exist.

For example, the crosscorrelation fit is more sensitive to the proper delay time than the proper amplitude. Moreover, the effect of the pP echo is expected to be larger than the spall echo's. Hence a search can be conducted for a pP time fit with some reasonable echo amplitude and the P_s echo can be ignored altogether.

With proper pP delays for two events established, the pP amplitudes are optimized next. These settings for pP will not vary appreciably when P_s search is considered later. Then the P_s search is conducted the same way: first in delay time and next in amplitude. Even so, the search procedure can be tedious.

Figure 10 shows the results of such a search for MAST and CHARTREUSE. The correlation coefficients (xmax) between the raw signals at RKON is .699. The best pP-only fit increased the xmax to .772 and occurred at 0.2 seconds for CHARTREUSE and 0.35 seconds for MAST. These times, which are rounded to the nearest 0.05 seconds, are equivalent to a velocity of 4.5 km/sec. Holding these pP delay times and echo amplitudes fixed, and then searching first for P_s delay times and then for P_s amplitudes, yielded an optimum correlation of .813 with a spall echo on MAST of 0.20 at 1.5 seconds, and no spall echo on CHARTREUSE.

The SCOTCH-SLED pair, shown in Figure 11, is another example of a waveform fit improved by this search technique. SCOTCH (3207 feet deep) and SLED (2393 feet deep) are separated by 3.8 kilometers. Even so, these raw signals recorded at RKON have a correlation of .646. The best fit after a pP-only deghosting search had a correlation of .785. However, the pP amplitude of -0.75 causes considerable ringing with the oscillations causing a chance alignment with the de-echoed SCOTCH waveform. With spall echoes added, a better fit is obtained with less ringing and a correlation peak of .821. Table VII summarizes correlation results from this optimum search for these pairs of events. Note that the relative distance to RKON is only 1 km.

The pP delay times for the best SCOTCH & SLED fits are longer than expected, which indicates a near surface velocity of less than 2 km/sec and delay times for SLED too close to those of SCOTCH, considering their relative depths. Even so, the pP and spall echoes have the expected signs and the spall echoes occur at reasonable delay times. The 1.75 seconds for the spall echo on SCOTCH was chosen to match Springer's (1974) report. The 1.20 seconds for a spall echo on SLED and the two amplitudes (+.35 for the SLED spall and +.20 for the SCOTCH spall) were found by optimum search. These results also show that the increase in correlation (xmax) with an optimum pP and a pP plus spall fit is about what would be expected from the synthetic echo degradation studies with KNICKERBOCKER and CHATEAUGAY. However, the parameters, which correspond to the optimum ones at RKON, do not improve the correlation at other sites for these same events. Also the SCOTCH pP and P_s delays for the SCOTCH vs. CAMEMBERT comparison do not agree with those found for SCOTCH vs. SLED (see Table VII).

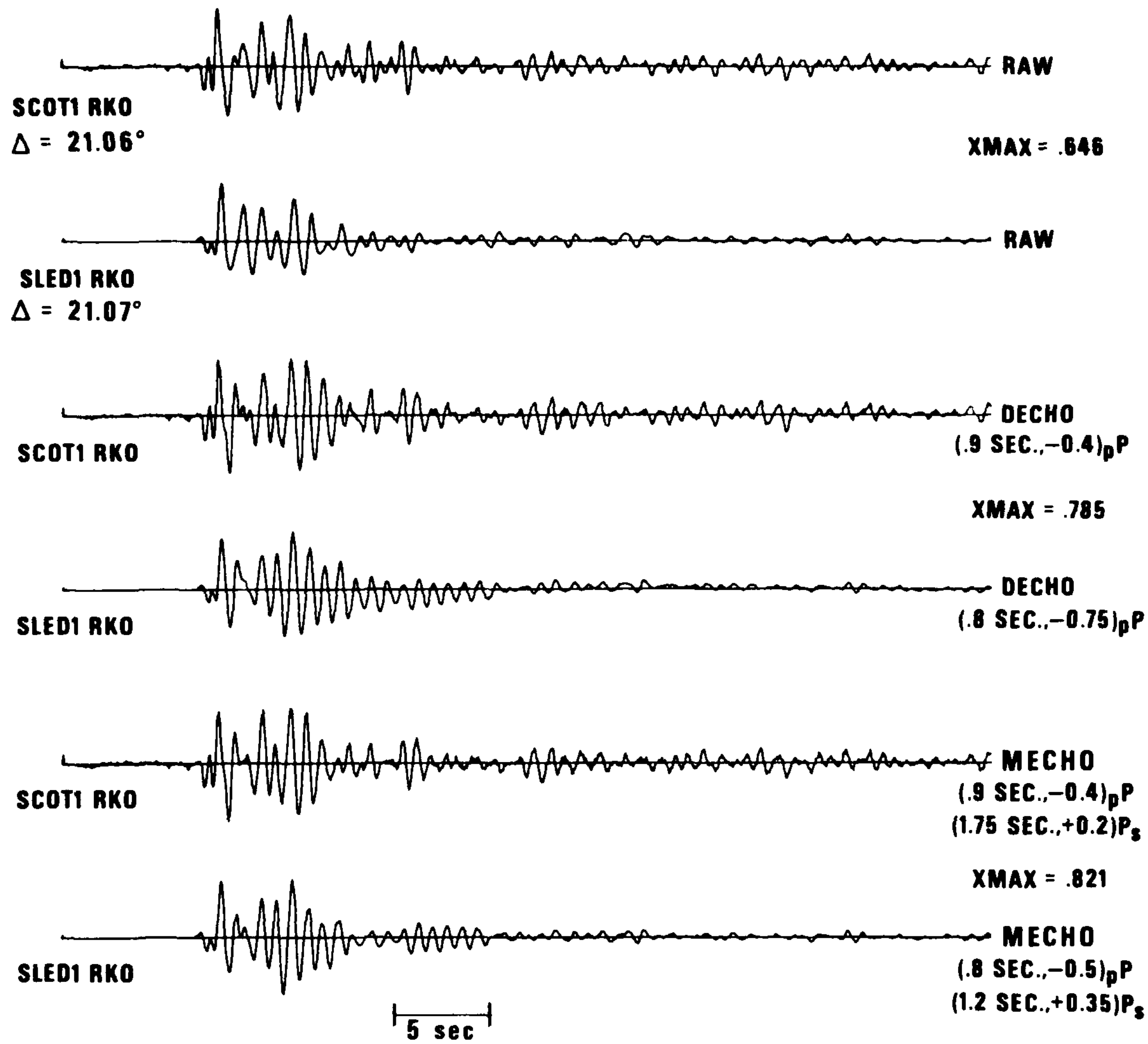

Figure 11. A deghosting search applied to SCOTCH and SLEDGE recorded at RKON. The raw signal correlation was .646; the best deghosting for pP only gave .785; the best deghosting for both pP and P_s gave .821.

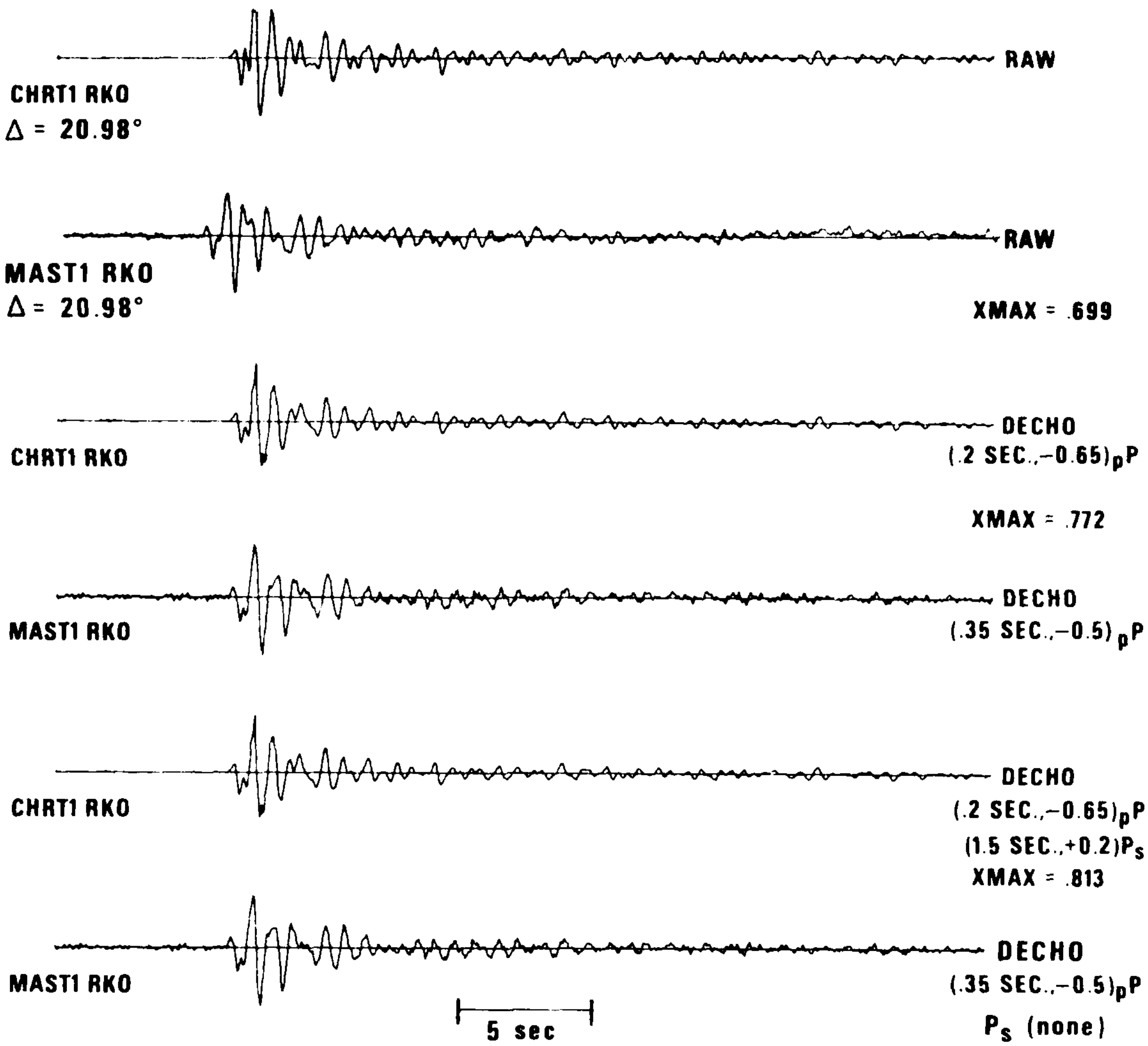

Figure 10. A deghosting search applied to nearby events. At RKON MAST and CHARTREUSE correlated at .699; the best deghosting for pP only gave a correlation of .772; the best deghosting for both pP and P_s gave .813.

TABLE VII

Correlation results at RKON from three pairs of Pahute Mesa Events.

Event Pair		
<u>MAST</u> (depth 2989 ft. Δ = 20.98°) vs.	<u>CHARTREUSE</u> (depth 2183 ft. Δ = 20.98°)	<u>xmax</u>
raw	raw	.699
deghost (.35 sec, -.5)	deghost (.2 sec, -.65) pP	.772
deghost (.35 sec, -.5) pP & (1.5 sec, + .2) P_s	deghost (.2 sec, -.65) pP	.813
<u>SCOTCH</u> (depth 3207 ft. Δ = 21.06°) vs.	<u>SLED</u> (depth 2392 ft. Δ = 21.07°)	
raw	raw	.646
deghost (.9 sec, -.4) pP	deghost (.8 sec, -.75) pP	.785
deghost (.9 sec, -.4) pP & (1.75 sec, + .2) P_s	deghost (.8 sec, -.5) pP & (1.2 sec, + .35) P_s	.821
<u>SCOTCH</u> (depth 3207 ft. Δ = 21.06°) vs.	<u>CAMEMBERT</u> (depth 4301 ft. Δ = 21.06°)	
raw	raw	.543
deghost (.50, -.3) pP	deghost (.70, -.3) pP	.579
deghost (.50, -.4) pP & (2.2, + 0.0)*P_s	deghost (.70, -.4) pP & (2.85, + .2) P_s	.617

* Zero spall amplitude means that the best correlation occurred with no spall echo on SCOTCH.

OTHER SEARCH METHODS

Several other methods were considered during attempts to ascertain the optimum echo parameters which would make the seismograms from a pair, or a set, of events, match. One method computed autocorrelations of the P-wave and coda at each station recording an event. Multiple path effects, like a source echoing function due to pP or spall, causes the autocorrelation function to possess significant peaks (echoes). The source autocorrelation echoes should persist over an entire network for one event whereas the path autocorrelation echo would not agree from station-to-station. Similarly, a second event autocorrelated over the same network should yield similar path echoes at each of the individual stations, but the source echoes, which persist over the network, should differ from those of the first event.

Both source echoes were detected, with a network-wide persistence and station echoes, and with an event-to-event persistence from the autocorrelation functions. However, the delay times detected, when used as parameters of the deghosting filters, did not produce signal waveforms which correlated better or as well as the original raw traces. This result persisted even when spectral whitening methods were used on the autocorrelations to cut down the monochromatic ringing and to emphasize the echoes.

With somewhat limited experimenting with cepstra analysis the information carried in the cepstra was also present on what appeared to be the whitened autocorrelation functions. The problem was trying to choose the proper delays from the many possibilities.

A deghosting method was tried in which the echo was a low-pass filtered version of the generating waveform, rather than an exact copy. Using this filtered deghosting program -(FECHO)- for the pP-only analysis of the SCOTCH-SLED pair, when using the optimum parameters from the optimum search at RKON, an improvement in correlations occurred (xmax's) at stations other then RKON. Furthermore, these pP-delay times matched those expected from the depth of the events. Rather than an impulse, spall echoes certainly could be low-pass filtered versions of the P-wave. Since not all of the spall material torn loose returns to the earth at the same time, the spallation material would generate an impulse smoothed out over time, which is a low-pass filtering effect. The pP echo could be similarly filtered if

the reflecting surface of the ground is irregular and not a plane. The high frequencies (shorter wavelengths) in the pP reflection would be scattered and attenuated which is, again, a low-pass filtering effect. Deghosting filters which do not account for this filtered effect when they should, could cause undesired ringing in the output (P-waveform estimation).

The computer time required poses the greatest problem in using the trial-and-error approach. The reason a filtered echo is not added to the multiple echo deghosting filters is that the MECHO program was already handling eight parameters, and another two to four would have further complicated an already tedious trial-and-error method. What is required is a more deterministic approach. Our model assumes that the teleseismic P-waveform is composed of a source effect and a path effect. For a pair of neighboring shots the path effects should match. The problem is to determine the source effect, including the echoes, so the presence of spall can be verified.

If two shots (one and two) are considered, then their SP-Z signals at a particular site will be designated as $x_1(t)$ and $x_2(t)$. Assume that the signal-to-noise ratio is large. If so, these two teleseismic signals, $x_1(t)$ and $x_2(t)$, are composed of

$$x_1(t) = w_1(t) * x_o(t)$$

$$x_2(t) = w_2(t) * x_o(t)$$

where the $w_i(t)$ is the source time function, including P, pP, and P_s waveforms for the ith event, and the $x_o(t)$ is the filter effect of the path. The symbol (*) means convolution so that if the source were a pure impulse (no echoes), the path effect $x_o(t)$ is the seismogram that would be recorded at this particular station. The spectral equations are

$$X_1(\omega) = W_1(\omega)\ X_o(\omega)$$

$$X_2(\omega) = W_2(\omega)\ X_o(\omega)$$

The ratio of the cross power spectrum of these two seismograms to the power spectrum of one of them would be

$$\frac{P_{12}(\omega)}{P_{22}(\omega)} = \frac{X_1X_2'}{X_2X_2'} \quad \frac{W_1X_o\ W_2'X_o'}{W_2X_o\ W_2'X_o'} = \frac{W_1W_2'}{W_2W_2'}$$

where the prime (') symbol means complex conjugate. This power spectral ratio (P_{12}/P_{22}) is stable and fairly straightforward to compute. Note, however, that the path effect, X_o, has divided out of the numerator and denominator. Moreover, if the spectrum of one of these source functions was approximated (W_2) then an estimate of the other one can be calculated.

Thus, $$\frac{P_{12}(\omega)}{P_{22}(\omega)} \cdot \hat{W}_2 = \frac{W_1W_2'}{W_2W_2'} \cdot \hat{W}_2 = \hat{W}_1$$

where the symbol (^) means approximation. The Fourier transform of this spectrum directly yields the echo pattern of the first event, $W_1(t)$. The one to use in the denominator would be the smaller event or the one which is expected to have no, or minimum, spall. Then a pP-only approximation of its echo pattern may be sufficient. If the approximation is incorrect, then the echo pattern of the result ($w_1(t)$) will not be a short finite set of echoes but rather a long, slowly decaying series. In this case, we re-estimate W_2 and try again. Thus, while the trial-and-error approach is not completely avoided, a gain is made in that only one function is approximated, and the result sought is a relatively simple one that can be recognized when correct.

This approach could also be used to identify the depth of a new event on the basis of an older one. In addition, the approach could expose complex or multiple shots based on such a comparison with an earlier, nearby event.

CONCLUSIONS

A deghosting approach has been followed in trying to detect spall signals on the teleseismic P-waveforms from two or more events with virtually the same source-to-receiver path. In studying Pahute Mesa explosions, two events, KNICKERBOCKER and CHATEAUGAY, within 0.5 kilometers of each other and within 25 meters of the same depth, produced almost identical seismograms at the same station. The correlation coefficient between these P-waveforms at RKON was 96%, a figure higher than between the SP-Z and SP-R from the same event.

For events with nearly the same location but different depths the P-waveforms though similar, show marked differences. Correlation coefficients between such P-waveforms normally varied between 50% and 70% at stations recording both events with high signal-to-noise ratios.

If no spall signals were present, the model assumed that deghosting corrections for the pP echoes alone ought to produce matching waveforms. Under pP deghosting the waveform fit improved, but in no case did a P-waveform pair correlating in the 50% to 70% range match the 96% found between KNICKERBOCKER and CHATEAUGAY.

Deghosting corrections for both pP and spall (P_s) echoes improved the waveform match over that of pP corrections alone. Again, for events with markedly different depths the pP and P_s, deghosting did not produce a waveform match as good as KNICKERBOCKER and CHATEAUGAY.

In all of these cases of nearby events for which pP and P_s deghosting was tried, the optimum matches occurred with the pP echo negative and the P_s echo positive, which were the polarities expected for these echoes.

The amount of improvement in the correlation coefficients from deghosting with pP to deghosting with both pP plus P_s echoes was similar to the correlation changes when KNICKERBOCKER and CHATEAUGAY (matching) seismograms were synthetically ghosted with spall echoes.

The amount of spall echo needed to achieve the optimum match was always equal to or less than 35% of the original P-waveform. The amount of P_s echo deghosting needed to achieve the optimum match would never change the

magnitude measurement appreciably (by more than 0.1 m_b) at that site.

The pP and P_s deghosting parameters, though they gave the best waveform corrections and best correlation coefficients at one site, did not always improve the waveform match or the correlation coefficients at the other site.

Because the correlation coefficients for the optimum matches were significantly less than 90%, more complex echo adjustments than our simple model allows may be necessary. Other differences in source mechanisms, such as strain release, may exist. We have already found that allowances for different corner frequencies improved the waveform and correlation match. Another possibility is low pass filtering for both the pP and P_s echoes. The corresponding optimum deghosting parameters for pP along, which had been determined by crosscorrelation search at RKON, improved the correlations and waveform match at other sites in the network, when low-pass filtering of pP echoes was considered.

Because pP and P_s deghosting improve the waveform match, but do not achieve a 90% correlation, a more deterministic approach is needed. The trial-and-error approach, even with an analyst using the graphics terminal on the computer, requires excessive computation time. The deterministic approach using cross power spectra as outlined in this report would calculate what actually was the echo model between two, thus accommodating filtered echoes or multiple spall.

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APPENDIX

Derivation of Deghosting Filters

APPENDIX: DERIVATION OF DEGHOSTING FILTERS

A deghosting filter (Sax, 1967) is the inverse of a filter which adds an echo. In time the weighting function of an echo filter would be

$$W_{echo}(t) = \delta(t) + a\delta(t-T) \tag{1}$$

where a is the echo amplitude relative to the initial signal, and T is the echo delay.

The frequency response of the echo filter is

$$W_{echo}(\omega) = 1.0 + ae^{-j\omega T} \tag{2}$$

and in terms of the Z-transform notation we could write it as

$$W_{echo}(Z) = 1.0 + aZ \tag{3}$$

where $Z = e^{-j\omega T} = z^n = (e^{-j\omega\Delta T})^n$ (4)

with the echo delay $T = n\Delta T$ or n times the sampling interval in digital systems.

The inverse to an echo filter is a de-echo or deghosting filter. In Z-transform notation

$$W_{decho}(Z) = \frac{1}{1.0 + aZ} = 1 - aZ + (aZ)^2 - (aZ)^3 + \ldots. \tag{5}$$

and in frequency

$$W_{decho}(\omega) = (1 + ae^{-j\omega T})^{-1} = 1 - ae^{-j\omega T} + a^2e^{-j2\omega T} - a^3e^{-j3\omega T} + \ldots \tag{6}$$

and in time

$$W_{decho}(t) = \delta(t) - a\delta(t-T) + a^2\delta(t-2T) - a^3\delta(t-3T) + \ldots \tag{7}$$

These inverse (decho) filters, which seem to require an infinite series of delays operating on the input can be made recursive. In Z-transform notation we have

$$W_{echo}(Z) = \frac{E_{out}(Z)}{E_{in}(Z)} = \frac{1}{1 + aZ}$$

clearing this equation we get

$$E_{out}(Z) = E_{in}(Z) - aZ\, E_{out}(Z)$$

This equation leads immediately to the recursive algorithm in terms of time with the continuous time variable, t, replaced by the discrete variable, $\Delta t, (m = 1,2,3,.....)$

$$E_{out}(m\Delta t) = E_{in}(m\Delta t) - aE_{out}((m-n)\Delta t)$$

Thus, the present value of the output, which is unknown, is calculated from the value of input and a past value of output, both of which are known. This algorithm is used in the DECHO program of the spall package on the PDP-15 computer.

For multiple echoes the algorithm is similar. For two echoes the multiple de-echo (MECHO) program uses

$$W_{mecho}(Z) = \frac{E_{out}(Z)}{E_{in}(Z)} = \frac{1}{1+a_1Z_1 + a_2Z_2} \tag{8}$$

$$E_{out}(Z) = E_{in}(Z) - a_1Z_1E_{out}(Z) - a_2Z_2E_{out}(Z) \tag{9}$$

and in time we have

$$E_{out}(m) = E_{in}(m) - a_1E_{out}\,(m-n_1) - a_2E_{out}(m-n_2) \tag{10}$$

where

$E(m)$ is $E(t)$ with $t = m\Delta t$. Here the derivation is analogous to the single echo case. The extension to more than two echoes follows directly.

For the filtered deghosting algorithm (FECHO) the frequency response of an impulse is inverted and followed by a filtered echo.

$$W(\omega)_{fecho} = \left(1 + b\frac{\alpha}{\alpha + j\omega} e^{-j\omega T}\right)^{-1} \tag{11}$$

Using a common Z-transform approximation[1] for $j\omega$,

$$j\omega \cong \frac{2}{\Delta t} \cdot \frac{1-z}{1+z} \tag{12}$$

and

$$e^{-j\omega T} = e^{-j\omega n\Delta t} = z^n \tag{13}$$

as before, we get

$$W(z)_{fecho} = \left(1 + \frac{\alpha\ b\ z^n}{\alpha + \frac{2}{\Delta t} \cdot \frac{1-z}{1+z}} \right)^{-1} = \frac{E(z)_{out}}{E(z)_{in}} \tag{14}$$

Now clearing to generate a polynomial in z times E_{out} equal to another polynomial in z times E_{in}, we generate a recursive algorithm for FECHO similar to those for DECHO and MECHO.

1. See any text on z-transforms.